THE GALACTIC FORCE AND STELLAR KINEMATICS PERPENDICULAR TO THE GALACTIC PLANE: O AND B STARS

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(Received 1963 August 15)

Summary

The component of the galactic force field perpendicular to the galactic plane in the solar neighbourhood is found by direct use of the definition of $K(z)$ in terms of the gradient of the kinetic energy and a plot of z against $\dot{z}$ for the O and B stars. This method yields a force law, linear to about $x = 200$ pc, that gives about the same magnitudes and period of oscillation as methods that have explicitly used star counts. The assumption of a gaussian velocity distribution in the effective galactic plane ($z = \pm 40$ pc) is found to be reasonable, and the most probable speed decreases linearly from 20 km/sec at $z = 0$ to 6 km/sec at $z = 200$ pc. O and B stellar distributions at various z are in accord with kinematical ages (time spent in orbit from the galactic plane), evolutionary ages, and the derived period of oscillation.

1. *Introduction.*—Now that a number of reliable determinations of the distances of O and B stars are available, investigations of their distribution in the Galaxy and aggregation in clusters have entered the literature (e.g. Blaauw 1956, Beer 1961). In general, studies of the proper motions and radial velocities of O and B stars have been concerned only with obtaining distances and cluster membership. In view of our present ability to obtain reliable extinction corrections and hence distances, particularly for the early-type stars, it seems worth while to study their galactic space motions and especially their z-motions (motions perpendicular to the galactic plane). It is the purpose of this paper to derive z-motions for representative O and B stars and to study kinematical ages and the z-force at low galactic heights ($z < 200$ pc) in the solar neighbourhood.

2. *Basic observational details.*—Whenever available, distances of O and B stars were obtained from published values of $(m_0 - M)$ or r if the investigator used extinction corrections obtained on the U, B, V system. Otherwise, the values of V and $B-V$ were reduced by assuming $A_V = 3E_{(B-V)}$ and using the intrinsic values of V and $B-V$ tabulated by Johnson & Iriarte (1958) and Johnson (1958), respectively. This procedure also made it possible to eliminate any faint OB subdwarfs. The following references were used: Beer (1961), Blaauw (1961, high-velocity stars), Feast *et al.* (1957), Hiltner (1956), Mendoza (1958), Oosterhoff (1951), Osawa (1959), Stebbins & Kron (1956), and Underhill (1955). Oosterhoff's colour indices can be transformed by formulae given by Beer. Several other sources gave small numbers of additional stars. In a very few cases we have assumed the extinction for a bright star to be zero if $B-V$ was not given.

The radial velocities were for the most part obtained from Wilson's catalogue (1953). Only determinations bearing qualities *a*, *b* and *c* on Wilson's scale were used. The same criterion held for stars in the Southern Hemisphere, whose radial velocities were given by Feast *et al.* (1955, 1957). All the proper motions were taken from Boss's *General Catalogue* (1936), except for a few stars with *Yale Catalogue* motions listed in Beer (1961). The Yale proper motions were converted to the GC system by the table in that catalogue (Schlesinger *et al.* 1934). The GC stars listed by Beer, however, were looked up again in the GC in order to obtain the probable errors not given by Beer.

Morgan & Oort (1951) pointed out that systematic errors as well as incorrect values of the Newcomb precession constants are inherent in the GC determinations. Moreover, the FK3 and N30 have been found to define better fundamental systems. We therefore transformed the GC proper motions into the FK3 system by the tables prepared by Kopff (1939). Corrections for the revision of the precessional constants were then made by the formulae given by Morgan & Oort (1951). The probable errors of the GC were assumed to remain the same.

The reductions of the observed radial velocities and proper motions to correct for solar motion and differential galactic rotation were carried out in the usual way, derived for example in the textbook by Smart (1938). The solar and galactic constants were taken to be:

$V_\odot = 20$ km/sec		(solar speed)
$\alpha_\odot = 18^{\mathrm{h}}\,1^{\mathrm{m}}\!\cdot\!8$	(1950)	(solar apex)
$\delta_\odot = +30^\circ\,3'\!\cdot\!0$	(1950)	
$\alpha_0 = 12^{\mathrm{h}}\,49^{\mathrm{m}}$	(1950)	(north galactic pole)
$\delta_0 = +27^\circ\,24'$	(1950)	
$A = +15$ km/sec/kpc		(Oort constants)
$B = -12$ km/sec/kpc		
$K = +2$ km/sec		(*K*-term)

The solar quantities are the values usually adopted for early-type stars (e.g. van de Hulst *et al.* 1954). The Oort constant B was obtained from our adopted A by assuming that the Sun is located 10 kpc from the galactic centre (Weaver 1961) and that the linear velocity of an object moving in a circular orbit at this distance is 270 km/sec. The galactic coordinate system in l^{II}, b^{II} was used (Lund 1961).

All the necessary reductions were carried out by use of an IBM 7090 with tables of proper motion corrections stored in the machine. Input data were the constants listed above and the following quantities for each star: α (1950), δ (1950), spectral class, luminosity class, $(m_0 - M)$, v_r (radial velocity), μ_α, μ_δ, and the maximum probable errors of the last four quantities. The output data contained: l^{II}, b^{II}, $r \cos b^{\mathrm{II}}$, z, V, $\dot{x}$, $\dot{y}$, $\dot{z}$, $z/\dot{z}$, and the maximum errors in the last seven quantities. These errors were estimated by augmenting each input datum by its probable error and obtaining maximum errors in the process of computation.

The output data were grouped according to quality based primarily on the size of the maximum error. Three useful classes seemed to form naturally, and they are listed in Table I. All other stars, not conforming to these three classes, were rejected. It should be mentioned that stars known to belong to clusters or streams, such as the Sco–Cen and Cas–Tau streams, were omitted. Of course, in most large lists of O and B stars some cluster members will almost certainly be present, since the O and B stars characteristically occur in groups.

TABLE I

Quality classes of stars used

Class	No. of stars	r (kpc)	v_r	p.e. μ''
A	32	$<0{\cdot}5$	a, b, c	$<0{\cdot}0015$
B	125	$<1{\cdot}5$	a, b, c	$<0{\cdot}005$
C	150	$<4{\cdot}0$	a, b, c	$<0{\cdot}01$

3. *Distribution of stars.*—In Fig. 1 we have plotted the distribution of O and B stars in galactic latitude and longitude. Clearly, the stars are well distributed in longitude and reasonably well in latitude. The tendency to cluster is evident in places, despite our efforts to avoid accepting obvious cluster members in our data. Nevertheless, these groups have few enough stars so as not to bias unduly the distribution. Open circles represent luminosity classes I–II, and filled circles luminosity classes III–V.

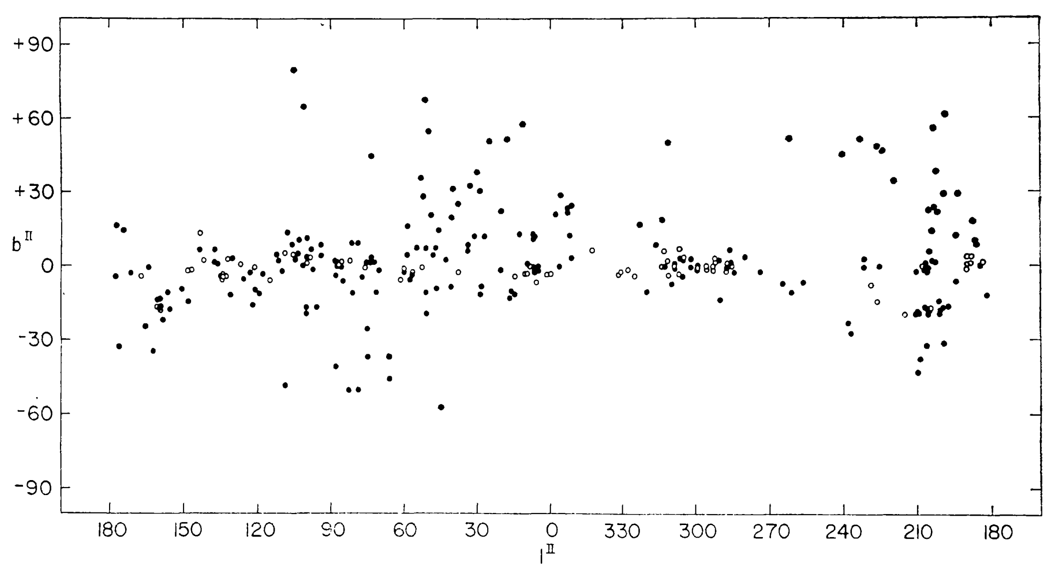

FIG. 1.—*Distribution of O and B stars in galactic latitude and longitude. Open circles refer to luminosity classes I–II and filled circles to classes III–V.*

The highly uniform character of the distribution of stars, even out to 1500 pc, is more meaningfully seen in Fig. 2, where we have plotted $r \cos b$ (distance from the Sun projected along the galactic plane) against z. Moreover, it is quite evident that the galactic plane is free from undue weighting; uniformity

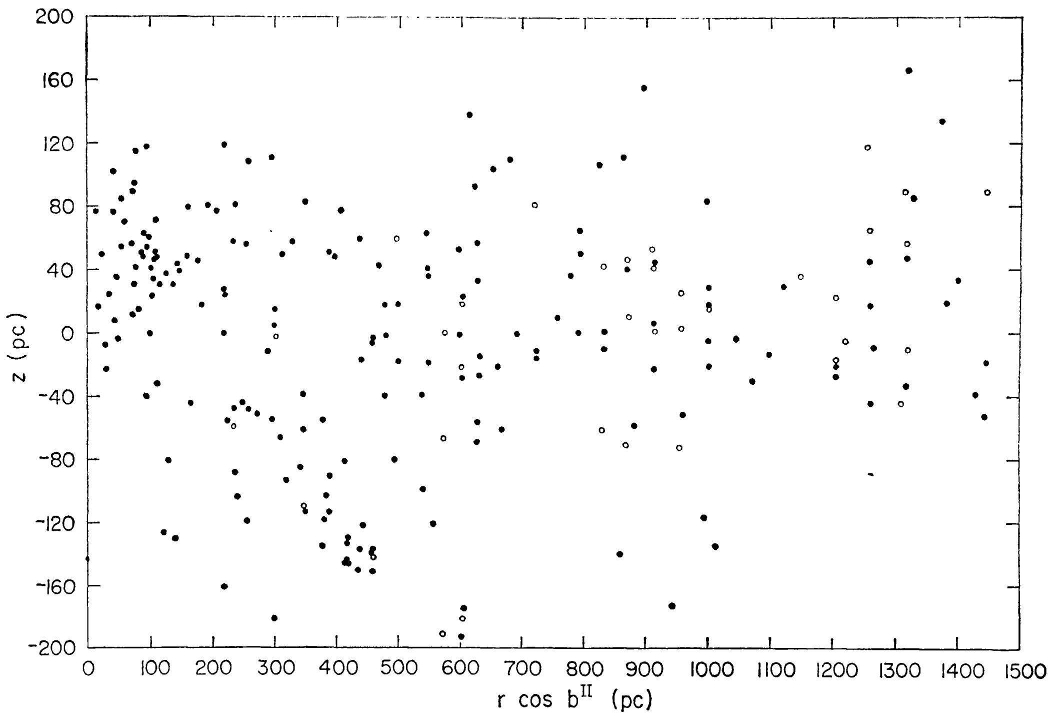

FIG. 2.—*Distribution of O and B stars as a function of height above the galactic plane and of distance from the Sun projected along the galactic plane. Same symbols as in Fig.* 1.

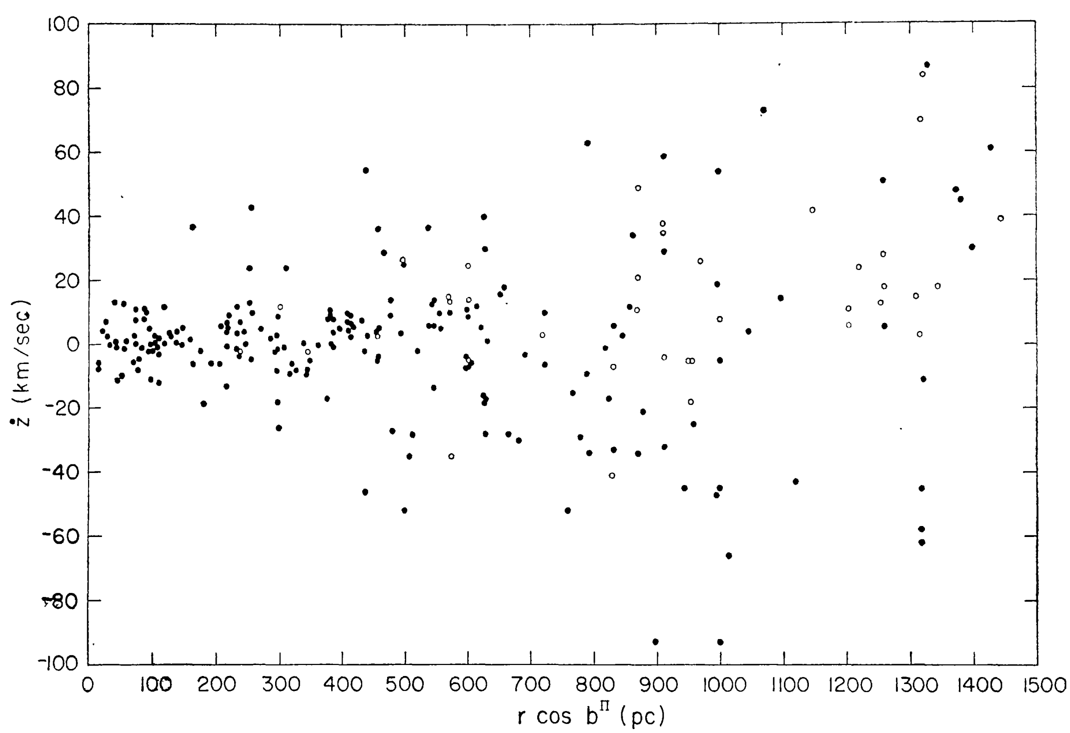

FIG. 3.—*Distribution of O and B stars in z-velocity space. Same abscissa and symbols as in Fig.* 2.

seems to characterize z as high as ± 100 pc. This means that we shall have a large enough sample of stars at high z to work with. The situation is fortunate, because the stars were chosen quite at random with no preference for any b^{II}. Although obscuration tends to make stars at low b^{II} unobservable, the effect should be negligible out to at least 500–1000 pc. In fact, it is only beyond this range where we see mainly OB supergiants (open circles). We conclude that this distribution of stars randomly selected is indeed representative of all the O and B stars.

Fig. 3 shows the distribution of $\dot{z}$ as a function of $r \cos b$. The distribution is seen to be almost rectangular out to 700 pc. At greater distances the errors in the measured proper motions and the approximations in the theory of galactic rotation make themselves apparent by widening the $\dot{z}$ distribution, for we assume that the physical processes causing these velocities are approximately the same at $r \cos b = 700$–1500 pc as at 0–700 pc. The main contributors beyond $r \cos b = 700$ pc were stars of the lowest quality class C. Therefore, in what follows we shall use only stars within this radius around the Sun in the galactic plane, which leaves us a working total of 168. About 25 per cent of these are of class C. The computed maximum errors of class C stars are comparable to those of classes A and B, but the errors are themselves often only slightly smaller or of the same magnitude as the quantities to which they refer. The errors are, however, extreme upper limits.

By restricting ourselves to $r \cos b < 700$ pc, we have the added advantage of probably excluding any blue Humason–Zwicky subdwarfs, even though they are not numerous at relatively low z. On the other hand, we limit our sample of the rare OB supergiants. At any rate, we may assume that the z-force will not change significantly over this small radius, an assumption supported by the theoretical work of Schmidt (1956).

4. *The z-component of the galactic gravitational field.*—Oort (1932) devised the following method for determining the component of the gravitational field perpendicular to the galactic plane, $K(z)$. Assuming a steady state in which $N(\dot{z})/N(\dot{z}_0) = \dot{z}_0/\dot{z}$, where N represents the number of stars at a given z and a zero subscript refers to the galactic plane, one can obtain the relation

$$N(\dot{z}) = \phi\left\{\sqrt{\left(\dot{z}^2 - 2\int_0 K(z)\,dz\right)}\right\} N_0,$$

where ϕ is some normalized velocity distribution. In order to make $N(\dot{z})$ separable in $\dot{z}$ and z, Oort made the reasonable assumption that ϕ was gaussian. Then it follows that

$$K(z) = \alpha^2 \frac{d \ln \Delta(z)}{dz},$$

where

$$\Delta(z) = \frac{N}{N_0} = \exp\left\{\frac{1}{\alpha^2}\int_0^z K(z)\,dz\right\}.$$

In this fashion Oort arrived at the first reliable determination of $K(z)$ from the observed z-density distributions of stars.

The most recent, and perhaps most thorough, determination of $K(z)$ was made by Hill (1960), who first derived a z-density distribution of the K giants, and then a $K(z)$ which he regards as reliable out to about 500 pc. Hill's paper also gives an exhaustive treatment of the history and theory of $K(z)$ computations by the z-density method.

An alternative approach to the calculation of $K(z)$ derives directly from the conservative nature of the gravitational field. The condition for a conservative field may be written

$$\mathbf{curl\ K} = 0$$

which implies that

$$\mathbf{K} = -\mathbf{grad}\ U,$$

where U is a scalar quantity that can be identified as the potential energy. Since the total energy per unit mass of a star can be written as

$$E = \tfrac{1}{2}V^2 + U = \text{constant},$$

we have

$$-\mathbf{grad}\ U = \mathbf{grad}\ \tfrac{1}{2}V^2,$$

where V is the speed. Hence we can write for the z-component of force

$$K(z) = \frac{d}{dz}\left(\tfrac{1}{2}\dot{z}^2\right) = \dot{z}\frac{d\dot{z}}{dz}.$$

If the functional relationship between z and $\dot{z}$ can be derived from a study of z and $\dot{z}$ of the members of a statistically reliable sample of stars, it is clear that $K(z)$ can be obtained. This is the procedure followed in the present work. To our knowledge, it is the first attempt at a $K(z)$ computation by this method.

In Fig. 4 we have plotted z against $\dot{z}$ for just those O and B stars from our survey that lie closer than $r\cos b = 700$ pc. The key fact to note is that the envelope describing the perimeter of the region of high density in the lower left portion of the figure also defines the functional form of the $\dot{z}$-dependence on z. The reason lies in the nature of the velocity distribution, which is assumed only to fall off fairly rapidly. If a star having some initial $\dot{z}$ leaves the galactic plane, then its $\dot{z}$ (whose functional relation with z depends on the force law) will diminish with increasing z, and the outer envelope of the $(z, \dot{z})$ plot should represent the orbit or curve of constant energy for a star near the high-velocity end of the initial velocity distribution. Clearly, from Fig. 4, the number of stars drops sharply at large z, and we can single out the orbit of a sample high-velocity star by selecting the perimeter of the envelope. Thus Fig. 4 represents essentially a large number of similar, concentric curves, whose density drops off with increasing z and $\dot{z}$.

It should be noted that the two methods of determining $K(z)$ are not independent. In Oort's method, $K(z)$ was determined by star counts at various z, assuming that with increasing z the number of stars decreases in accordance with the acting force. In the present method we use criteria depending explicitly on the two-dimensional phase space: a decrease of total star numbers as z increases, *and* a decrease in $N(\dot{z})$ as $\dot{z}$ increases, at a given z.

Oort's method also depends on $\dot{z}$, however, in that the velocity dispersion, α, occurring in his equation for $K(z)$ must be obtained in some way from observations of $\dot{z}$. Formally, his equation reduces to $\dot{z}\,d\dot{z}/dz$, but his method has the disadvantage (for small numbers of stars) of not dealing directly with the observed $(z, \dot{z})$ and of requiring two basic assumptions: (1) a steady state, and (2) an explicit formula for the velocity distribution, separable in z and $\dot{z}$.

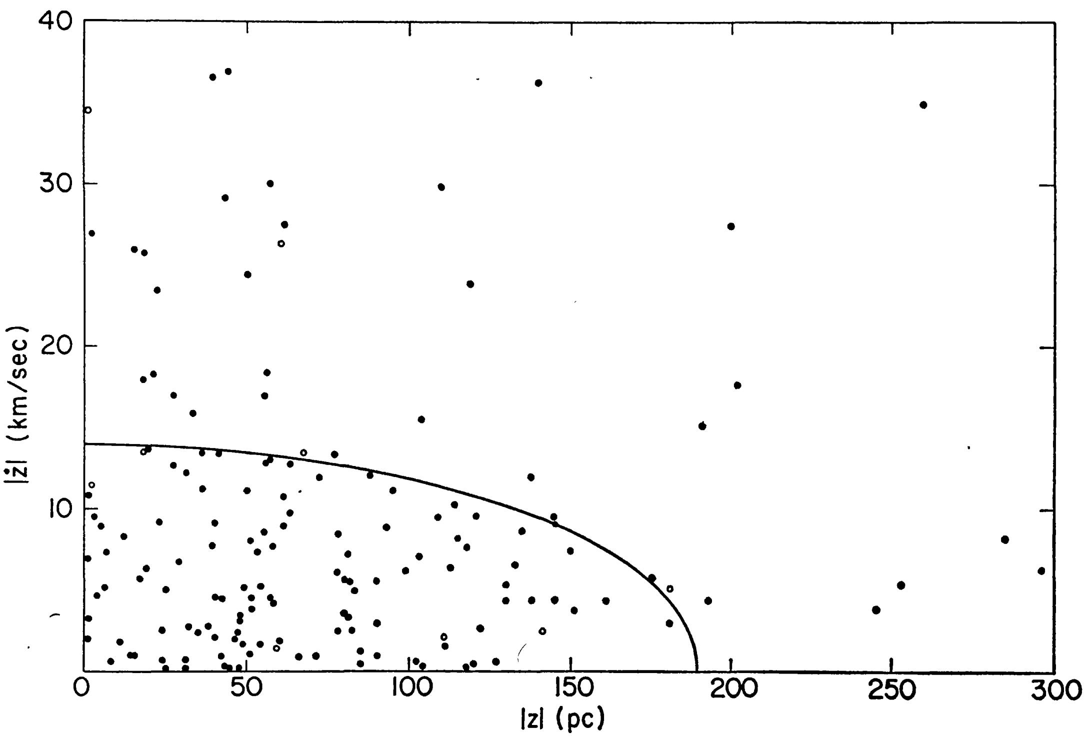

FIG. 4.—*Plot of the O and B stars in z phase-space. The curve bounds the region of greatest density.*

The straightforward way to determine the z, $\dot{z}$ dependence in Fig. 4 would be to draw the best curve fitting the envelope of points and having a slope of zero at $z=0$ and an infinite slope at $\dot{z}=0$. Since the force operates in the same way on a star leaving the galactic plane or entering it, we have reflected the stars with negative $\dot{z}$ about the z-axis. Moreover, we assume that the stars are symmetrically located above and below a uniform galactic plane, and have similarly reflected stars with negative z about the $\dot{z}$-axis; this assumption was justified by separate plots of $+z$ and $-z$ against $\dot{z}$.

If the envelope around the plotted points were sufficiently well defined, a high-order polynomial could be fitted to the empirical curve to obtain $\dot{z}(z)$. Then the force would be obtained directly from $\dot{z}\,d\dot{z}/dz$. Finally, a least-squares solution for n in $K(z)=z^n$ would be made from values calculated from the polynomial representation. This procedure, however, could not in the present case be adopted, in view of the uncertainty of how to draw the curve around the plotted points—our statistics are simply not numerous enough.

The approach we have taken, therefore, is the following. The form of the function $\dot{z}(z)$ has been calculated for force laws of the type $K(z)=z^n$. As displayed in Table II, the results show that for $n<-1$ the figures are open;

physically, these values of n are meaningless close to the galactic plane. The case $n=-1$ yields logarithmic forms for the potential and $\dot{z}(z)$, and therefore is also rejected. The only closed figures are represented by $-1<n<\infty$. (We note that a diagonal line across Fig. 4 yields a force that is the sum of a constant, attractive component and a replusive component proportional to z.)

TABLE II

Functional forms of $\dot{z}(z)$ for various force laws (α and β constant)

Power n	Force law	Coefficient of force law	$\dot{z}(z)$	Figure of $\dot{z}(z)$
∞	∞	...	...	Indeterminate
>-1	z^n	$-\frac{n+1}{2}\alpha^2$	$\alpha\sqrt{(\beta-z^{n+1})}$	Closed
...	$k+z$	$-\alpha^2\beta$, α^2	$\alpha(-\beta+z)$	Diagonal line
-1	z^{-1}	$-\frac{1}{2}\alpha^2$	$\alpha\sqrt{(\beta-\ln z)}$	Semi-infinite
<-1	z^n	$\frac{n+1}{2}\alpha^2$	$\alpha\sqrt{(z^{n+1})}$	Open
$-\infty$	0	0	α	Horizontal line

In Fig. 5 we have drawn curves for the cases $n=0$, 1, and 2. For larger n, the curve becomes more convex. Superposition of these curves on to Fig. 4 gave a value of n between 0 and 2. For the sake of simplicity and in the light of our knowledge of previous work, we chose $n=1$ to plot on Fig. 4; this yields an ellipse.

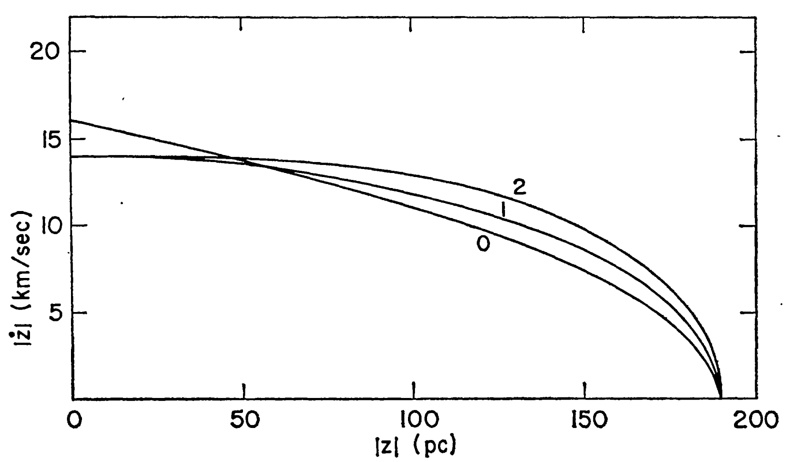

FIG. 5.—*The functional relation between $\dot{z}$ and z is plotted for various force laws dependent on a power of z. Curves are labelled with the power n (cf. Fig. 4).*

The minor axis is quite clearly close to $b=14$ km/sec. The major axis is less well defined, but apparently lies somewhere between $a=170$ and 190 pc. Actually, the upper limit seems also to be the best fit; therefore we have adopted

$z \leqslant 200$ pc as the domain of the O and B stars in our sample. An analytical expression for the orbital ellipse takes the obvious form

$$\frac{\dot{z}^2}{b^2} + \frac{z^2}{c^2a^2} = 1,$$

where b and $\dot{z}$ are expressed in km/sec, z in km, a in pc, and $c = 3{\cdot}086 \times 10^{13}$ km/pc. By a simple differentiation of this equation with respect to z, we obtain

$$K(z) = \dot{z}\frac{d\dot{z}}{dz} = -\frac{b^2}{c^2a^2}z.$$

A linear restoring force implies simple harmonic motion, with a particular solution given, for example, by $z = z_{\max} \sin \omega t$, where $\omega = 2\pi/P$ is the frequency and P the period of oscillation. In general, $\ddot{z} = -\omega^2 z$, so that we have

$$P = (2\pi c)\frac{a}{b}\ \text{sec} = 6{\cdot}144 \times 10^6 \frac{a}{b}\ \text{years}.$$

Using this expression, we may compute the period for $a/b = 170/14$ and $190/14$, obtaining

$$0{\cdot}75 \times 10^8 \leqslant P \leqslant 0{\cdot}83 \times 10^8\ \text{years}.$$

This is to be compared with the value $0{\cdot}84 \times 10^8$ obtained by Oort (1932). For $K(z)$ we have, with z expressed in pc,

$$K(z) = \begin{array}{ll} -2{\cdot}20 \times 10^{-11}z\ \text{cm/sec}^2 & (a/b = 170/14) \\ -1{\cdot}76 \times 10^{-11}z\ \text{cm/sec}^2 & (a/b = 190/14). \end{array}$$

In Table III our values are compared with those of Oort (1932) and Hill (1960). Our upper and lower limits are in surprisingly close agreement with the values given by Oort and Hill, respectively. The agreement of our $K(z)$ and P with their values implies that the O and B stars are little affected by any viscosity that would be produced from an incomplete dissociation from the interstellar medium.

Table III

Galactic z-force in the solar neighbourhood

z (pc)	$K(z) \times 10^9$ (cm/sec^2)			
	OB (this paper)		(Oort 1932)	gK (Hill 1960)
50	−0·880	−1·10	−0·880	−1·36
100	−1·76	−2·20	−1·74	−2·75
150	−2·64	−3·30	−2·57	...
200	−3·52	−4·40	−3·43	−5·14

5. *Velocity distributions*.—It was assumed in the above discussion and is evident from Fig. 4 that the distribution of velocities at any given z falls off sharply. In order to interpret the data more quantitatively, we make the following three assumptions: (1) stars are born in the galactic plane, (2) the $\dot{z}$ distribution in the galactic plane is gaussian and the space speed, V, is maxwellian, and (3) stars leave the galactic plane after a certain period of time that is constant for all stars and independent of $\dot{z}$. Observed locations of OB clusters and associations directly support assumption (1) that these stars are born close to the

galactic plane. Assumption (2) seems to be supported by the histograms in Fig. 6 for $0 < z < 40$, which we define to be the galactic plane. Assumption (3) will be discussed in the following section.

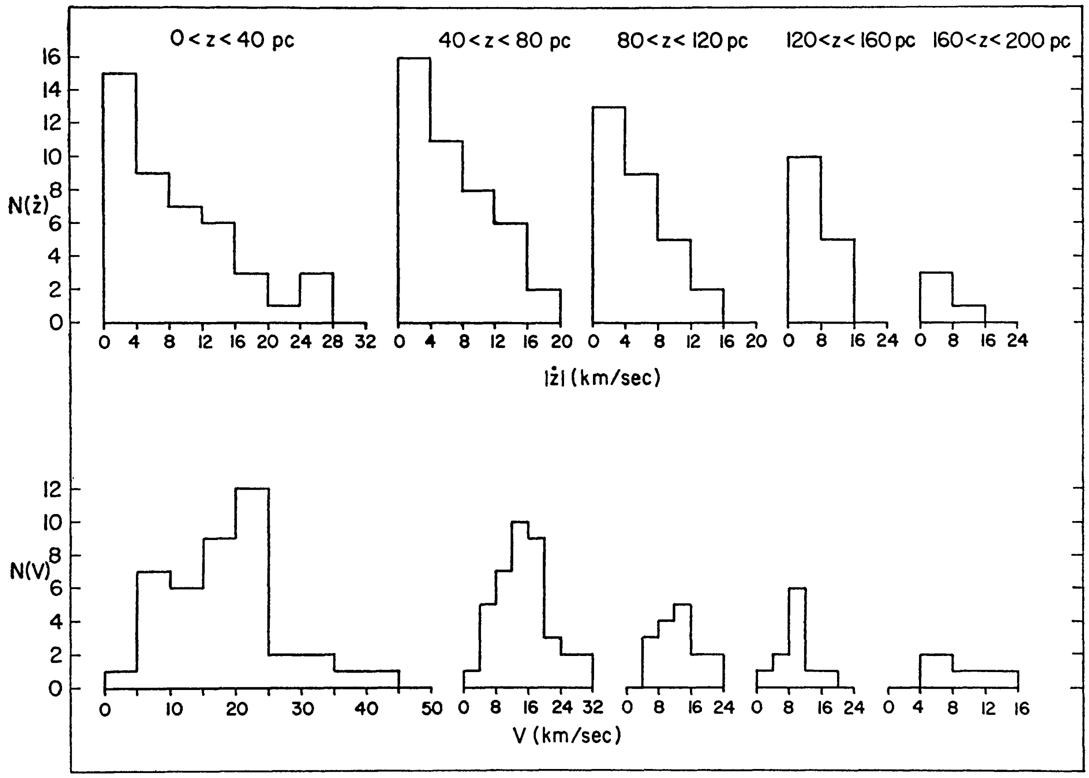

FIG. 6.—*Histograms of the number of stars with given z-velocity and space speed, V, are constructed for various intervals of z.*

Oort (1932) showed that, regardless of the form of the force law, $K(z)$, an initially gaussian velocity distribution in the galactic plane remains gaussian at higher z, as a result of the separability of z and $\dot{z}$. Using his equation for $N(\dot{z})$ as given in the previous section and a linear force law, $K(z) = -\omega^2 z$, we obtain

$$N(\dot{z}) = \frac{N_0}{\alpha\sqrt{\pi}} f(z)\, e^{-\dot{z}^2/\alpha^2},$$

where $f(z) = \exp(-\omega^2 z^2/\alpha^2)$. The distribution is clearly gaussian (relative to the initial N_0 in the plane) up to a multiplicative factor, $f(z)$. Integration of $N(\dot{z})$ over $\dot{z}$ yields the total number of stars at given z, assuming a steady state:

$$N = N_0 f(z).$$

Since the initial $\dot{x}$, $\dot{y}$ distributions are unchanged, we have for the distribution of speeds at any given z

$$\frac{N(V)}{N_0} = \frac{f(z)}{\alpha^3 \pi^{3/2}} 4\pi V^2\, e^{-V^2/\alpha^2}.$$

This equation has the same maxwellian form as the V distribution in the plane. However, stars with initial $V_0 = \dot{z}_0 < \omega z$ will not reach z. Therefore the effective α will be $\alpha_0 - \omega z$ or α decreases linearly with z.

We now compare these results with the observational data. For stars at a given z, the velocity distribution of stars entering the galactic plane should be the same as that for stars leaving the plane, regardless of the relative numbers of stars involved. We therefore constructed histograms of the number of stars with $\dot{z}\downarrow$ (incoming) and $\dot{z}\uparrow$ (outgoing) for various intervals of z. The two series of histograms were much the same, so that we combined the two star groups in Fig. 6 in order to build up our data. Even though the histograms at high z have little weight, the distributions are very similar at all distances out to $z=200$ pc and may be considered gaussian.

The numbers of stars expected at given z under the assumption of a steady state are given in the third column of Table IV. N_0 is here chosen to be the observed N_0. The deviation from the observed numbers at high z is discussed in the next section.

TABLE IV

OB stellar data as functions of height above the galactic plane

z (pc)	N_{obs} (↑↓)	N_{exp} (↑↓)	$\dfrac{N_{obs}(\uparrow)}{N_{obs}(\downarrow)}$	α_{obs} (km/sec)	α_{exp} (km/sec)
0– 39	48	48	1·1	20	20
40– 79	52	44	1·7	16	15
80–119	31	31	0·6	12	12
120–159	17	10	0·2	9	9
160–199	7	0	(1·3)	6	6
0–199	155	...	1·0	...	...

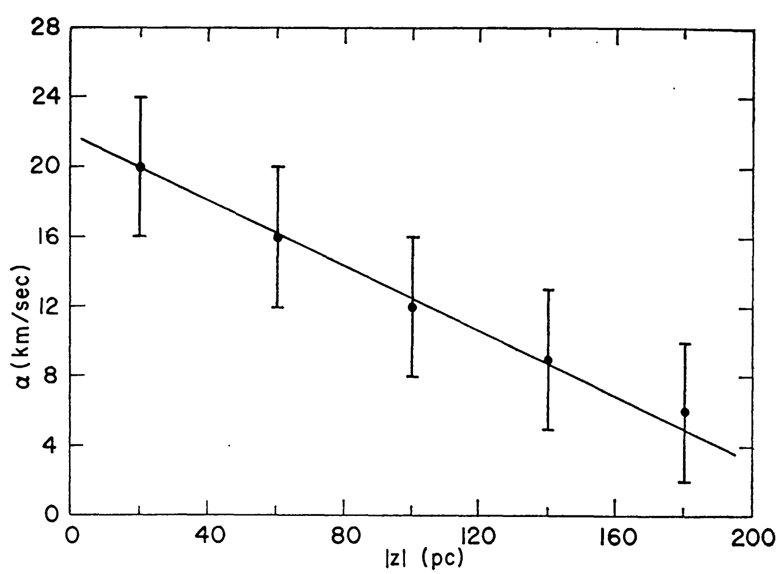

FIG. 7.—*The most probable speed, α, is plotted against z from the results of Fig.* 6 *for N(V).*

The most probable speed at a given height, $\alpha(z)$, can be found from histograms of the number of stars with given space speed, V. The lower histograms in Fig. 6 show reasonably well the maxwellian nature of the distribution. Inspection yields $\alpha=20$ km/sec in the galactic plane. At higher z, α decreases linearly until at $z=200$ pc, α becomes 6 km/sec. The relation is shown in Fig. 7, and comparison with the expected α ($=\alpha_0-\omega z$) is made in Table IV. The agreement is very close.

6. *Kinematics.*—With the derived force law and the evidence that the orbital amplitudes of the O and B stars do not much exceed 200 pc (the limit of the derived linear force law), we can calculate $\dot{z}(z)$ and therefore the time spent in orbit from the galactic plane, which we define to be the kinematical age,

$$\tau_{\text{kin}}(z) = \int_0^z \frac{dz'}{\dot{z}(z')} = \frac{1}{\omega} \tan^{-1}\left(\frac{wz}{\dot{z}}\right).$$

In Table V $\langle \log \tau_{\text{kin}} \rangle$ is computed from a mean $\log(z/\dot{z})$ for the stars in three spectral intervals.

Presumably the OB supergiants evolve from main sequence dwarfs of roughly the same spectral type, and it would be interesting to compare their kinematical ages with those of the dwarfs. However, the data do not warrant averaging τ_{kin} for the supergiants alone, and we must consider together the stars of all luminosity classes.

Table V

Mean z-quantities and ages by spectral type for all luminosity classes

Sp	No. of stars	$\langle z\uparrow \rangle$ (pc)	$\langle \dot{z}\uparrow \rangle$ (km/sec)	$\log \tau_{\text{kin}}$ (in yrs)	$\log \tau_{\text{ms}}$ (in yrs)	No. of stars	$\langle z\downarrow \rangle$ (pc)	$\langle \dot{z}\downarrow \rangle$ (km/sec)
O5–O9·5	6	31·7	12·5	6·7	7·0	5	133·0	7·2
B0–B4	32	77·4	10·7	6·9	7·6	33	116·0	6·2
B5–B9·5	23	85·0	5·4	7·1	8·3	20	48·0	7·9

In Table V the main sequence lifetimes are those listed by Schwarzschild (1958), with an extrapolated age for the O stars. Since the orbital quarter-period is 2×10^7 years, it is clear that the O stars will end their active lives even before reaching their apices. But most of the O stars for which we have data are of spectral class O9·5 and their $\tau_{\text{ms}} = 2 \times 10^7$ years. Then some of them could complete slightly more than a quarter-oscillation. The last two columns of Table V show that indeed we do find some, at high z, moving down toward the galactic plane. The data seem to suggest that at least the O–B4 stars begin to leave the galactic plane shortly after birth. This conclusion lends support to assumption (3) in the previous section.

Since exit and return to the galactic plane should take $\sim 4 \times 10^7$ years, the O9·5–B4 stars that are moving out of the plane should be doing so for the first time. Hence we might expect the O stars to have a lower $\langle z \rangle$ than the early B stars, since the O stars are more massive and their initial velocity distribution in the galactic plane falls off more rapidly. Although the late B stars will have completed two to five oscillations during their active lives, it is significant that their $\langle z \rangle$ is even greater than that for the O–B4 stars.

The relative numbers of all the stars entering and leaving the galactic plane are listed in the fourth column of Table IV, for successive intervals of z. At low z there seems to be an excess of stars leaving the plane, as we expect, but at high z more are returning than leaving. Moreover, the distribution of stars expected from the assumption of a steady state falls off sharply at $z > 119$ pc, whereas the observed distribution does not. What, if anything, this means is uncertain, unless two disturbances or periods of star-birth, separated in time by about one quarter-oscillation, disrupted the steady state in the recent past.

7. *Discussion.*—It is of interest to recall that the maximum orbital amplitude for the O and B stars in the solar neighbourhood is about 200 pc. This corresponds to the strictly linear portion of the force law, as determined by Hill (1960). At $z > 200$ pc, the power n changes considerably according to his work on the K giants. This circumstance explains why the ellipse, which corresponds to a linear force law, is fairly well defined on Fig. 4. It also partly explains the small scatter near $z = a$, although thinning numbers of stars also contribute. In this connection we note that stars of higher velocity would mask the form of the force law near $z = a$, as the ellipse would go smoothly over into some other geometrical figure corresponding to the force law valid at $z > a$. Thus using stars of higher velocity to map out a composite force law for all z renders determination of the forms of $\dot{z}(z)$ at just low and at just high z impossible. However, if enough stars are available to obtain a good, completely empirical $\dot{z}(z)$ curve, a polynomial fit could yield discrete values of the force to the highest observed z.

Although the O and B stars are ideally suited for the accurate determination of the force up to $z \sim 200$ pc, it is clear that stars of increasingly higher mean z and $\dot{z}$ can map out higher z-regions of the force. Table IV shows that even among the O and B stars themselves gradations of z and $\dot{z}$ are evident. Hence the stars of lower mass and higher $\langle z \rangle$, in other words, later in the spectral sequence, are necessary for extending the determination of $K(z)$ to higher z. At these higher z, the considerations of the exponent of the force law as presented in Table II will take on more importance.

Acknowledgments.—The authors wish to thank Dr Morton Roberts for suggesting this problem and also to thank him and Dr David Layzer for critically reading the manuscript. It is a pleasure to acknowledge further helpful criticism from Dr. Andrew Young. We should like to thank Dr. Robert Jastrow for his hospitality at the Institute for Space Studies, New York.

Harvard College Observatory,
Cambridge,
Massachusetts:
1963 *August.*

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